Transverse Mixing Coefficients in the North Saskatchewan River

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by

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for

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ABSTRACT

More rigorous regulation and monitoring of effluent discharges in Alberta has created a demand for additional information on the variation of the transverse mixing coefficient with changes in the hydraulic characteristics in the North Saskatchewan River. A number of mixing studies were done in this river in the past; a search of existing data uncovered twelve tracer dye studies with sufficient information to evaluate the transverse mixing coefficient. Various types of experiments and methods of analysis were used to evaluate transverse mixing coefficients ranging between 0.02 m^2/s and 0.34 m^2/s for river discharges ranging between 59 m^3/s and 202 m^3/s.

Two dimensionless parameters were identified: the dimensionless transverse mixing coefficient, k_t and the dimensionless diffusion factor, β. Both these parameters showed no significant correlation with hydraulic characteristics for data reported in the literature. However, a trend was identified between the actual mixing coefficient and the discharge for the lower bound data from the North Saskatchewan River downstream of Edmonton. Relationships between the dimensionless parameters and discharge were derived from this trend using the Leopold-Maddock relationships which describe the variation of the hydraulic characteristics with discharge. However, it was found to be just as accurate, for the range of discharges tested, to use mean values of $k_t = 0.25$ or $\beta = 0.0004$ to describe the mixing rate. Both parameters can not be constant at the same time so, if k_t is assumed constant, a second relationship between β and discharge is obtained. Additional mixing experiments at higher discharges are needed to determine which of these relationships best describes the variation of β with discharge.

The data indicates that the river slope and hydraulic roughness of a river reach play a role in determining the value of the transverse mixing coefficient. As well, reaches with high sinuosity tend to have much higher mixing rates. The data also seemed to indicate that the presence of an ice cover increased the transverse mixing coefficient which is contrary to the results in the literature.

ACKNOWLEDGEMENTS

This report could not have been completed without the efforts of all the people involved in the original tracer dye experiments. As well, S. Lovell, S. Stanley, Dr. N. Rajaratnam, and Dr. D. Smith of the University of Alberta helped to trace some of the reports. The staff of the Standards and Approvals Division of Alberta Environment also help to identify some of the experiments by searching their records for Letter's of Permission. The cross-section information needed for the ARC calibrations was tabulated by B. Trevor.

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1.0 INTRODUCTION

1.1 Background

The North Saskatchewan River has been used for the disposal of both industrial and municipal wastes for many years. Increased awareness and knowledge of the environmental impacts of many of the wastes has led, in recent years, to more rigorous regulation and monitoring of effluent discharges into the North Saskatchewan River. This, in turn, has created a demand for more accurate information about the mixing characteristics of the river.

In the last twenty years, a number of mixing experiments have been performed on the North Saskatchewan River between Edmonton and the border with Saskatchewan. Most of these experiments have been done to define the mixing characteristics of a specific river reach for a specific client; however, mixing characteristics can vary considerably with location, discharge and ice conditions. No comprehensive analysis of mixing data exists for this river.

Planning Division of Alberta Environment recently had a mixing model called MULTI developed for them by HydroQual Consultants Ltd to assist them in planning and regulating effluent discharges into the North Saskatchewan River (McDonald et al., 1987). Some calibration has been done for this model but additional data was deemed necessary to improve the confidence in the model results. Rather than spend the considerable sums necessary to measure the pertinent information, it was decided to determine whether the information available from previous tests was sufficient to define the mixing characteristics of the river.

In 1990, the Alberta Research Council's Surface Water Engineering Group measured the travel times and longitudinal dispersion characteristics of the North Saskatchewan River between Edmonton and the Saskatchewan border for both open water and ice covered conditions. The transverse mixing characteristics were not evaluated at that time; however, it has since been determined that this information was required as well. The data from these tests can be analyzed further to evaluate the transverse mixing coefficients in these reaches. Since the Alberta

Research Council already possessed this data as well as some data from earlier experiments, the Council was requested by Alberta Environment to perform a comprehensive evaluation of transverse mixing characteristics of the North Saskatchewan River.

1.2 Objectives

The objective of this report is to define as well as possible the transverse mixing characteristics of the North Saskatchewan River by

1. searching the literature for transverse mixing information on the North Saskatchewan River;

2. estimating the transverse mixing coefficients from previous longitudinal dispersion experiments by calibrating a two-dimensional dispersion model; and

3. comparing the transverse mixing coefficients obtained from the above sources with the hydraulic characteristics at the time of the measurements.

2.0 TRANSVERSE MIXING THEORY

The fundamental three-dimensional mass transport equation can be simplified for applications in the two-dimensional mixing zone by integrating the equation over the depth so that the terms representing advective and diffusive transport in the vertical direction are eliminated (Holley et al.,1972). The resulting equation is

$$[1] \qquad \frac{\partial}{\partial t}(hC) + \frac{\partial}{\partial x}(huC) + \frac{\partial}{\partial z}(hwC) = \frac{\partial}{\partial x}\left(he_x\frac{\partial C}{\partial x}\right) + \frac{\partial}{\partial z}\left(he_z\frac{\partial C}{\partial z}\right)$$

where C, u and w are the depth-averaged concentration, streamwise velocity and transverse velocity, h is the local depth, and e_x and e_z are the longitudinal and transverse mixing coefficients respectively.

The complete two-dimensional equation given above can be simplified further. If steady state conditions are assumed, the time derivative term can be eliminated. The second derivative of concentration with longitudinal distance can also be eliminated because it is small compared to the first derivative. As well, the cartesian coordinate system used to express the above equation is cumbersome to use in natural channels so a curvilinear coordinate system was introduced by Yotsukura and Sayre (1976). This coordinate system follows the river planform and introduces the metric coefficients, m_x and m_z to account for variations in longitudinal and transverse distances caused by river curvature. Equation [1], therefore, becomes

$$[2] \qquad \frac{\partial}{\partial x}(m_z huC) + \frac{\partial}{\partial z}(m_z hwC) = \frac{\partial}{\partial z}\left(\frac{m_x}{m_z}he_z\frac{\partial C}{\partial z}\right)$$

This equation can be solved numerically in its present form; but, in order to obtain an analytical solution, a coordinate transformation must performed using the cumulative discharge defined by

$$[3] \quad \Sigma q = \int_0^z (m_z h u)\, dz$$

Yotsukura and Sayre (1976) performed this coordinate transformation and obtained the following equation

$$[4] \quad \frac{\partial C}{\partial x} = \frac{\partial}{\partial q}\left(u h^2 m_x e_z \frac{\partial C}{\partial q}\right)$$

This equation can be simplified further if the diffusion factor D_z is defined by

$$[5] \quad D_z = \frac{1}{Q}\int_0^Q (u h^2 m_x e_z)\, dq$$

The diffusion factor, D_z, can also be defined in terms of the average flow parameters (Beltaos, 1979)

$$[6] \quad D_z = \psi U H^2 m_x e_z$$

where U, H, m_x, and e_z are cross-section average values and ψ is a shape velocity factor defined as follows

$$[7] \quad \psi = \int_0^1 \left(\frac{h}{H}\right)^3 \left(\frac{u}{U}\right)^2 d\left(\frac{z}{B}\right)$$

where B is the river width and z is the transverse location. The value of ψ varies from 1.0 for a rectangular channel to 3.2 for extremely non-uniform natural channels. Values of ψ calculated

from data reported by Yotsukura and Cobb (1972) were found to vary between 1.2 and 2.9, the same range as reported by Beltaos (1979).

The analytical solution of equation [4] for a point source located at the bank is

$$[8] \qquad C = \frac{2\frac{\partial M}{\partial t}}{Q\sqrt{2\pi\xi}}\left[\exp\left(-\frac{\eta^2}{2\xi}\right) + \sum_{m=1}^{\infty}\left[\exp\left(-\frac{(2m-\eta)^2}{2\xi}\right) + \exp\left(-\frac{(2m+\eta)^2}{2\xi}\right)\right]\right]$$

where $\partial M/\partial t$ is the injected mass per unit time; Q is the total discharge; $\eta = \Sigma q/Q$, the dimensionless discharge; $\xi = 2xD_y/Q^2$, a dimensionless distance, and m is an integer (usually less than 9) which accounts for the reflections from the opposite bank (Elhadi et al., 1984).

3.0 MIXING EXPERIMENTS

The data from a number of tracer dye experiments on the North Saskatchewan River were gathered by collecting reports from sources such as private consultants and the University of Alberta as well as from data files at the Alberta Research Council. Reports describing some of the experiments were obtained through personal contacts but the existence of other studies was discovered because the experiments required Letters of Permission from the Standards and Approvals Division, Alberta Environment before they could be undertaken. A search of the files back to 1977 by staff from the Standards and Approvals Division turned up 7 Letters of Approval which uncovered a total of 12 individual tracer dye experiments for which transverse mixing data was available. A brief description of each of these experiments is given in table 1.

The hydraulic characteristics of the test reaches are given in table 2. While river slopes and roughnesses were fairly consistent for most of the experiments, the discharges varied between 59 m^3s and 202 m^3s. This range of discharges is representative of low flow conditions but no information exists for higher flows.

3.1 Steady State Experiments

For the steady state tests, Rhodamine WT dye solutions were injected continuously using a constant head apparatus to maintain a constant injection rate. Three steady state tests have been performed on the North Saskatchewan River, one by the Alberta Research Council and two by the University of Alberta (Table 1). The ARC test, done in 1977, covered a distance of 19.9 km between Cloverbar and Vinca Bridge (Hwy 38). The two UofA tests, done in 1990, covered short distances downstream of the two Edmonton water treatment plants, the E.L. Smith plant just upstream of Edmonton and the Rossdale plant just downstream of the Walterdale bridge in Edmonton (Milne, 1991). All three of these steady state injections were done near the banks rather than in the centre of the channel.

Table 1 Tracer dye experiment descriptions.

Organization	Reach Location	Date	Length (km)	Injection Type	Injection Location
HydroQual	Quesnell Bridge to u/s of Groat Bridge	Aug. 1987	5.0	Unsteady	30m from Right Bank
HydroQual	Quesnell Bridge to u/s of Groat Bridge	Aug. 1987	5.0	Unsteady	15m from Left Bank
HydroQual	Millwoods outfall to Quesnell Bridge	Aug. 1987	5.0	Unsteady	Centre of channel
University of Alberta	Ft.Saskatchewan to Vinca Bridge	Feb. 1972	21.5	Unsteady	Right Bank
University of Alberta	Downstream of Rossdale WTP	Sept. 1990	4.5	Steady State	Left Bank
University of Alberta	Downstream of E.L. Smith WTP	Sept. 1990	3.1	Steady State	Left Bank
Alberta Research Council	Sturgeon River to Redwater River	Oct. 1977	19.9	Steady State	8m from Right Bank
Alberta Research Council	Cloverbar to Waskatenau	Oct. 1977	74.5	Unsteady	30m from Right Bank
Alberta Research Council	Cloverdale Br. to Ft Saskatchewan	Mar. 1990	36.2	Unsteady	3 points across channel
Alberta Research Council	Cloverdale Br. to Ft.Saskatchewan	Oct. 1990	36.3	Unsteady	3 points across channel
Alberta Research Council	Shandro Bridge to Duvernay Bridge	Mar. 1990	51.3	Unsteady	2 points across channel
Alberta Research Council	Shandro Bridge to Duvernay Bridge	Oct. 1990	50.4	Unsteady	2 points across channel

Table 2 Hydraulic characteristics during the tracer dye experiments.

Reach Location	Date	Discharge (m^3/s)	Slope	Velocity (m/s)	Top Width (m)	Depth (m)	Manning Roughness
Quesnell Bridge to u/s of Groat Bridge	Aug. 1987	199	0.00040	0.56	200	1.80	0.053
Quesnell Bridge to u/s of Groat Bridge	Aug. 1987	202	0.00040	0.56	200	1.80	0.053
Millwoods outfall to Quesnell-Bridge	Aug. 1987	200	0.00010	0.66	198	1.50	0.020
Ft.Saskatchewan to Vinca Bridge	Feb. 1972	59	0.00038	0.29	216	0.94	0.041
Downstream of Rossdale WTP	Sept. 1990	197	0.00025	0.76	158	1.63	0.029
Downstream of E.L. Smith WTP	Sept. 1990	146	0.00054	0.73	145	1.38	0.039
Sturgeon River to Redwater River	Oct. 1977	190	0.00042	0.62	213	1.60	0.045
Cloverbar to Waskatenau	Oct. 1977	142	0.00044	0.59	156	1.54	0.047
Cloverdale Br. to Ft Saskatchewan	Mar. 1990	129	0.00036	0.59	172	1.14	0.035
Cloverdale Br. to Ft.Saskatchewan	Oct. 1990	124	0.00036	0.66	170	1.19	0.032
Shandro Bridge to Duvernay Bridge	Mar. 1990	120	0.00036	0.50	218	1.12	0.026
Shandro Bridge to Duvernay Bridge	Oct. 1990	152	0.00036	0.58	210	1.16	0.036

3.2 Unsteady Experiments

A number of unsteady tracer dye experiments have been performed over the length of the North Saskatchewan River. Only eight of these experiments had sufficient information reported to determine the transverse mixing characteristics relative to the hydraulic characteristics. Three of these experiments covering a reach between the Millwoods storm sewer outfall to about one kilometre upstream of Groat Bridge were done in 1987 by HydroQual Consultants Ltd. for the City of Edmonton (UMA Engineering, 1988). Another experiment was done in February, 1972 between Ft. Saskatchewan and Vinca Bridge by the University of Alberta (Pospisilik, 1972). The other five instantaneous or "slug" injection tests were done by the Alberta Research Council. One of these tests was done in 1977 between Cloverbar and Waskatenau in conjunction with the steady state test done that year (Beltaos and Anderson, 1979). The other four ARC slug tests were all done in 1990. Two tests were done in March under ice conditions; one injection from the Cloverdale pedestrian bridge in Edmonton and the other injection at the Shandro bridge near Wasel (Van Der Vinne, 1991a). However, the reach between Edmonton and Ft. Saskatchewan was ice free due to thermal effluent discharged in the reach. Two other dye injections were done at these same two locations in the fall for low flow open water conditions (Van Der Vinne, 1991b). Transverse mixing coefficients can be estimated from these slug tests by using the test data to calibrate two dimensional mixing models.

4.0 TRANSVERSE MIXING COEFFICIENTS

4.1 Evaluation of Coefficients

Transverse mixing coefficients can be evaluated using a number of different methods depending on the nature of the test and type of data gathered. Some methods use analytical solutions of equation [4] to evaluate the coefficient while other methods use numerical solutions. Analytical evaluations tend to be easier to compute but they usually assume uniform hydraulic characteristics over the test reach. The error introduced by this assumption increases as the length of the test reach increases. Numerical solutions, however, use actual hydraulic characteristics but take more effort to evaluate the coefficient. Input data to numerical models must also satisfy certain numerical stability criteria to obtain valid solutions. This means that either considerable effort is required to define the channel characteristics or much of the channel shape must be interpolated from the available data. Therefore, no matter which method is used to evaluate the transverse mixing coefficient there will be some uncertainty in the result.

4.1.1 Analytical Evaluations

Two analytical methods have been used to evaluate the transverse mixing coefficient. The method used depends on whether the data has been obtained from a steady state concentration experiment or from an unsteady concentration experiment.

4.1.1.1 Steady State Concentrations

The diffusion factor can be evaluated analytically using the relationship given by Gowda (1984)

$$[9] \qquad D_z = \frac{\sigma_q^2}{2x}$$

where σ_q^2 is the variance of the concentration distribution with respect to the cumulative discharge. Once this diffusion factor is known and the channel shape factor is estimated, the transverse mixing coefficient can be calculated from the test data by rearranging equation [6]. This procedure was performed to obtain the mixing coefficient given in table 3 for the steady state experiment done by Alberta Research Council staff in October, 1977 between the Sturgeon River and the Redwater River.

This method of evaluating the transverse mixing coefficient is probably the most simple and straight forward of the methods described in this report, however, there are some limitations. The results of these steady state tests are usually very site specific and apply over short reaches. Long distance steady state tests are not practical due the volume of dye required to maintain the steady state condition. Also, because of the injection equipment involved, the injection sites tend to be near the banks where there is greater variability in transverse mixing due to changes in channel shape.

4.1.1.2 Unsteady Concentrations

The diffusion factor, D_z , can also be estimated analytically for unsteady or time varying concentration experiments using a technique developed by Beltaos and Anderson (1979). The diffusion factor is calculated from

$$[10] \quad D_z = \frac{\sigma_\theta^2}{2\int_0^x f(x)dx}$$

where σ_θ^2 is the variance of the dosage distribution across the channel rather than of the concentration distribution. This dosage is defined as

Table 3 Mixing characteristics of the tracer dye experiments.

Reach Location	Date	e_z (m^2/s)	k_t	D_z (m^5/s^2)	D_zB/Q^2	B/H	U/U*
Quesnell Bridge to u/s of Groat Bridge	Aug. 1987	0.036	0.24	0.13	0.00066	111	6.7
Quesnell Bridge to u/s of Groat Bridge	Aug. 1987	0.10	0.67	0.37	0.0018	111	6.7
Millwoods outfall to Quesnell Bridge	Aug. 1987	0.040	0.70	0.12	0.00059	132	17.2
Ft.Saskatchewan to Vinca Bridge	Feb. 1972	0.093	2.36	0.048	0.0029	230	6.9
Downstream of Rossdale WTP	Sept. 1990	0.34	3.29	1.37	0.0056	97	12.0
Downstream of E.L. Smith WTP	Sept. 1990	0.053	0.45	0.15	0.0010	105	8.5
Sturgeon River to Redwater River	Oct. 1977	0.033	0.25	0.068	0.00040	133	7.6
Cloverbar to Waskatenau	Oct. 1977	0.0216	0.17	0.060	0.00047	101	7.2
Cloverdale Br. to Ft Saskatchewan	Mar. 1990	0.02	0.28	0.031	0.00032	151	9.3
Cloverdale Br. to Ft.Saskatchewan	Oct. 1990	0.02	0.26	0.037	0.00041	143	10.2
Shandro Bridge to Duvernay Bridge	Mar. 1990	0.033	0.66	0.041	0.00063	195	11.2
Shandro Bridge to Duvernay Bridge	Oct. 1990	0.025	0.34	0.039	0.00036	181	9.1

$$[11] \quad \theta = \int_0^\infty C\,dt$$

and the function, f(x), is defined as

$$[12] \quad f(x) = 1 - (1-\eta_0)\frac{Q}{M}\theta_{RB} - \eta_0\frac{Q}{M}\theta_{LB}$$

where η_0 is the fraction of cumulative discharge at the centroid of the dosage distribution across the channel. The transverse mixing coefficient given for the 1977 slug test between Cloverbar and Waskatenau was obtained in this fashion. The difficulty with this method is that it not only requires sufficient sampling across the channel but also requires sampling over time as the dye cloud passes the sample site. Unfortunately, the four ARC unsteady experiments done in 1990 could not be analyzed in this manner because the dosage distributions across the channels were not measured in sufficient detail to calculate the dosage variances accurately.

4.1.2 Numerical Evaluations

The calibration of numerical models with actual measurements can also be used to evaluate transverse mixing coefficients. However, care should be taken when comparing the coefficient resulting from a numerical model with those of a different numerical model or an analytical solution. The transverse mixing coefficients used to calibrate numerical models may include effects of the numerical formulation or grid spacing. When a particular model is being used to model the mixing in a river, the transverse mixing coefficient may require some adjustment so that the model reproduces the calibration data more closely.

To perform the calibrations, flow information such as discharges and cross-sections obtained for each of the tracer dye experiments were input into the model along with the injection and sample site locations. The models were then run with a number of different values

of mixing coefficient. The coefficient which produced the best approximation of the peak concentration and distribution across the section was assumed to be the average mixing coefficient for that reach.

4.1.2.1 MULTI Model Calibrations

The transverse mixing coefficients obtained by HydroQual Consultants Ltd. were evaluated by using test data to calibrate a numerical model called MULTI (UMA Engineering, 1988). This model uses a dimensionless parameter β to adjust the rate of transverse mixing. This dimensionless parameter is defined by the following function

$$[13] \qquad \beta = \frac{D_z B}{Q^2}$$

The MULTI model uses an analytical solution of equation [1] for steady state conditions similar to equation [8] but with a finite time interval and an additional term to account for the decay of a non-conservative material. The complete solution for a time-varied spill is obtained by summing the results of a number of short duration steady state intervals. One minor drawback of the MULTI model is that it uses the mean velocity of the river in all the stream tubes so it does not predict the lag of the near bank streamtubes compared to the centerline streamtubes. This may produce a significant source of error when concentration distributions from multiple effluent sources are superimposed.

4.1.2.2 ARC Model Calibrations

The Alberta Research Council model calibrations were done using a model developed by Beltaos and Arora (1988). This model is unique in that it uses a variable grid spacing along the streamtubes to minimize numerical diffusion. Only one coefficient, the transverse mixing

coefficient, is needed to describe the transverse as well as the longitudinal mixing. The variable grid spacing allows the differential advection, or the variation in velocity across the channel, to be modeled correctly. This differential advection is the mechanism which produces the much more rapid longitudinal mixing observed in most two dimensional time-varied experiments.

Some limitations of Beltaos and Arora's model made it possible to evaluate transverse mixing only in the first reach downstream of each injection. One of these limitation is that the amount of computing power required increases exponentially with river length. The computation time and the memory size are functions of the number of grid elements and the number of time steps, both of which increase as the reach length increases. This means that there is a practical limit to the reach lengths that can be modelled using this approach. Another limitation of the existing model is that it has not been designed to accept time-varied concentration inputs. With some effort, the model can be enhanced to include this capability but this has not yet been done. Because the model could not accept time-varied concentration inputs, the test results could not be analyzed on a reach by reach basis. That is, only one transverse mixing coefficient can be obtained for each injection.

Some imprecision exists in the transverse mixing coefficients obtained from these model calibrations because the experiments were not designed to measure the transverse mixing. The transverse concentration distribution was defined by only three samples taken across the river and the discharge distributions across the sections were estimated from the local flow depths. Only the first sample site in each test reach was used to evaluate the transverse mixing coefficients due to the computational constraints of the model.

4.1.2.3 TRSMIX Calibrations

The TRSMIX model developed by Putz et al. (1984) numerically models steady state mixing in a natural channel using the streamtube concept. The model also has the capability to model first order decay of pollutants. TRSMIX was used by Milne (1991) to estimate transverse mixing coefficients for two steady state experiments done by the University of Alberta's Civil

Engineering Department. The values given in table 3 are averages of the values given for each test because Milne used a different mixing coefficient to model each measured distribution rather that finding the mixing coefficient which best fit all the distributions.

4.2 Hydraulic Context

The mixing coefficients obtained from a particular tracer dye experiment are not only site specific but are also only applicable for the discharge and velocity which occurred during the tests. A relationship should be established between the various parameters to make the measured transverse mixing coefficients relevant for other locations and flow conditions.

Dimensional analysis indicates that the transverse mixing coefficient can be made dimensionless by dividing it by flow depth and shear velocity (Fischer et al., 1979)

$$[14] \qquad k_t = \frac{e_z}{HU_*}$$

According to Elhadi, et al. (1984), the dimensionless transverse mixing coefficient should be a function of the aspect ratio B/H, the relative roughness, U/U*, and the stream sinuosity. However, the test data available from natural rivers shown in figures 1 and 2 indicates that the dimensionless coefficient is virtually independent of the aspect ratio and relative roughness, at least for the range of values shown.

The dimensionless diffusion factor, β used by the model MULTI is related to the dimensionless mixing coefficient, k_t by

$$[15] \qquad \beta = \frac{HU_*}{BU} \psi m_x k_t$$

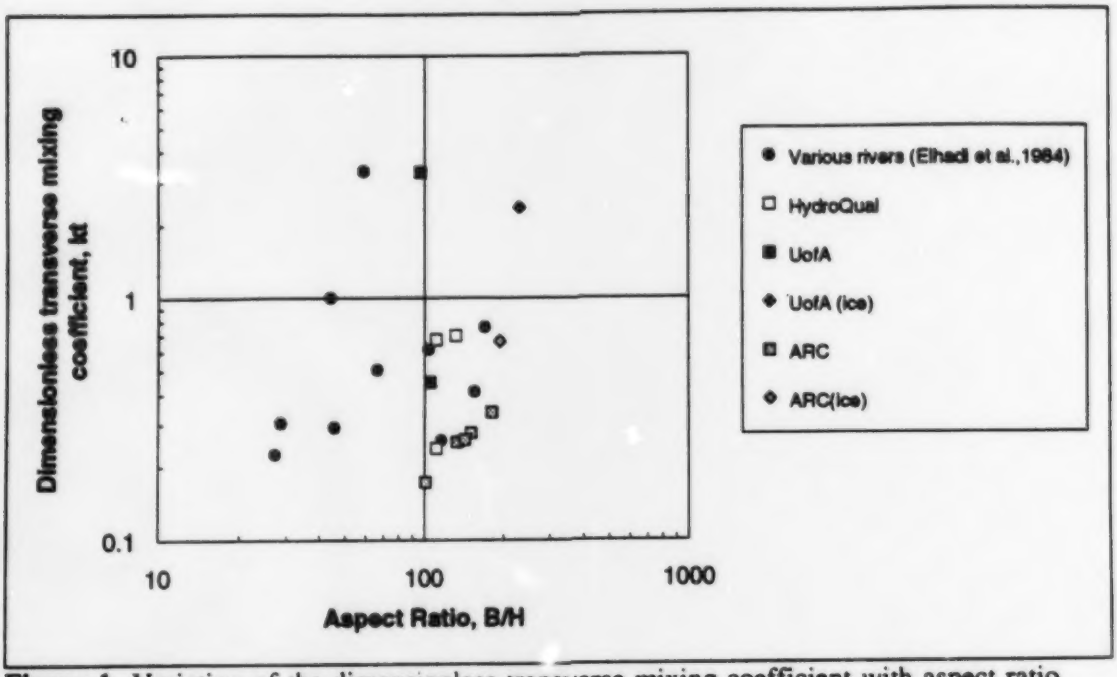

Figure 1 Variation of the dimensionless transverse mixing coefficient with aspect ratio.

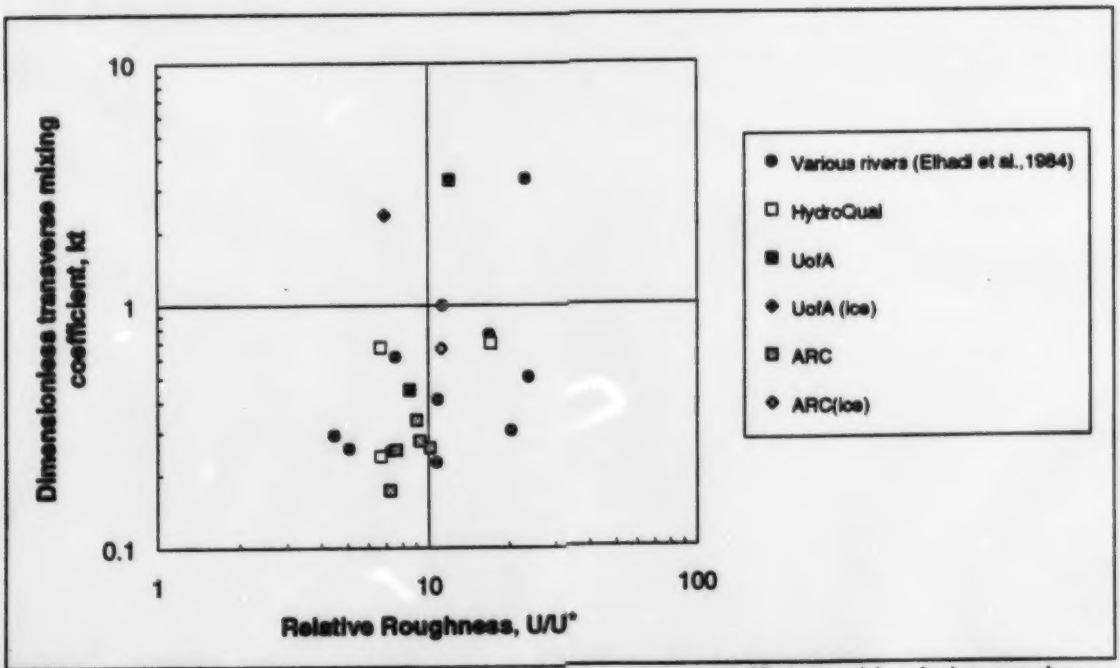

Figure 2 Variation of the dimensionless transverse mixing coefficient with relative roughness.

Just as with k_t, no significant correlation can be seen between β and river aspect ratio or relative roughness (figures 3 and 4).

The scatter exhibited by the data from the relatively straight reaches shown in figures 1 and 2 is significant but concentration distributions are relatively insensitive to changes in the mixing coefficient. For example, increasing e_z from 0.025 to 0.040 in the ARC model produced less than a 10% change in the peak concentrations at a particular location (figure 5). This insensitivity also applies when the coefficients are being evaluated; the evaluated coefficients are likely to only have an accuracy of one digit. Therefore, for the range of the available test data, even if the dimensionless transverse mixing coefficient is treated as a constant, there will be little or no reduction in the accuracy of the models, at least for relatively straight reaches.

4.2.1 Discharge

No distinct trends in the complete data set could be established in the previous section using dimensional analysis, but when e_z is plotted against discharge, a distinct trend can be seen in the lower bound of the North Saskatchewan River data (figure 6). This lower bound data comes from the tests where there was little or no influence of river curvature or ice cover on the mixing coefficient. If the relationship between transverse mixing coefficient and discharge is assumed to be log-linear, the resulting best fit function becomes

$$[16] \qquad e_z = 0.00005\, Q^{1.24}$$

This function was evaluated for discharges between 120 m^3/s and 190 m^3/s. It is not known whether this relationship is valid outside this range.

The good fit of equation [16] with this subset of the data may be due to chance. If the relationship is real, it follows that a relationship should also be found between k_t and Q because both H and U_* are also functions of Q. When the Leopold-Maddock equations (Ray and

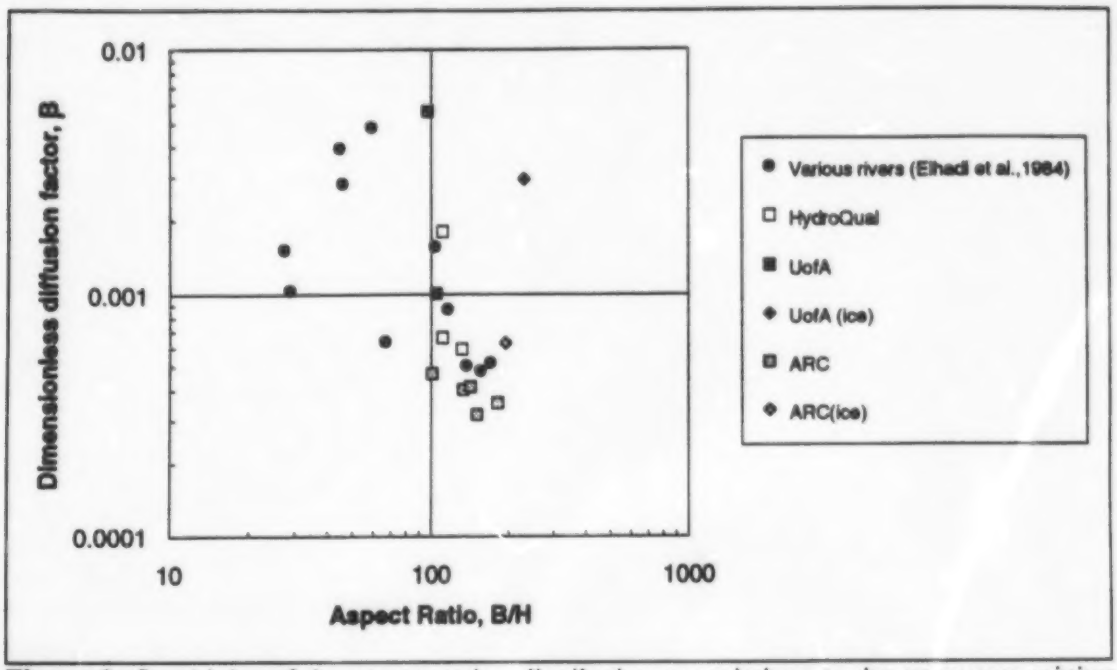

Figure 3 Sensitivity of the concentration distribution to variations in the transverse mixing coefficient.

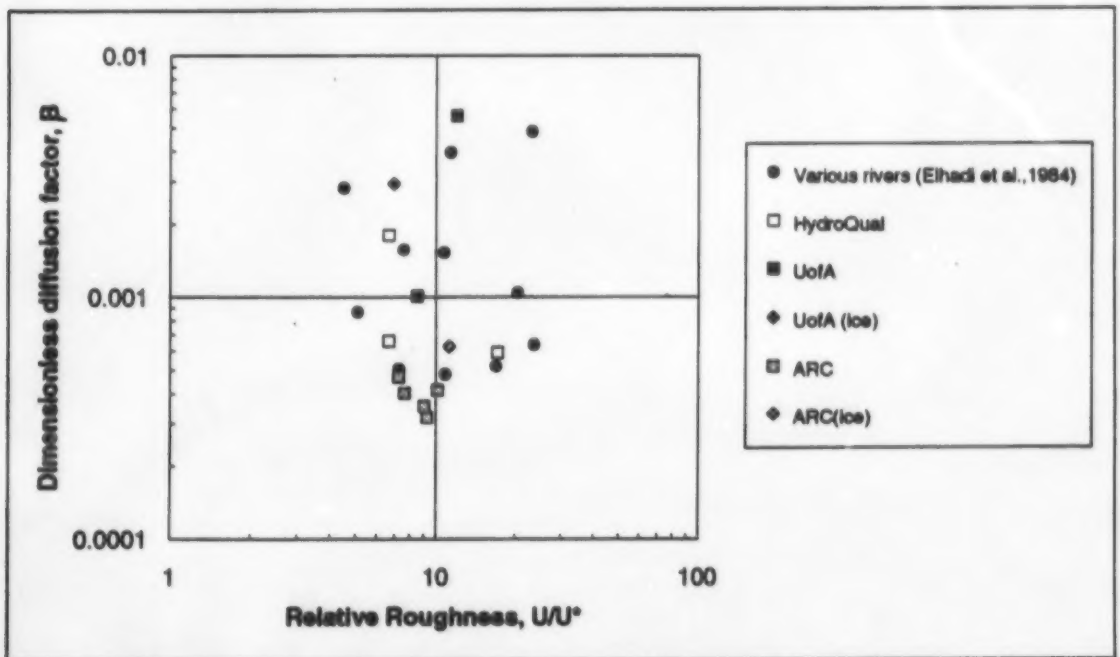

Figure 4 Variation of the transverse mixing coefficient with discharge in the North Saskatchewan River.

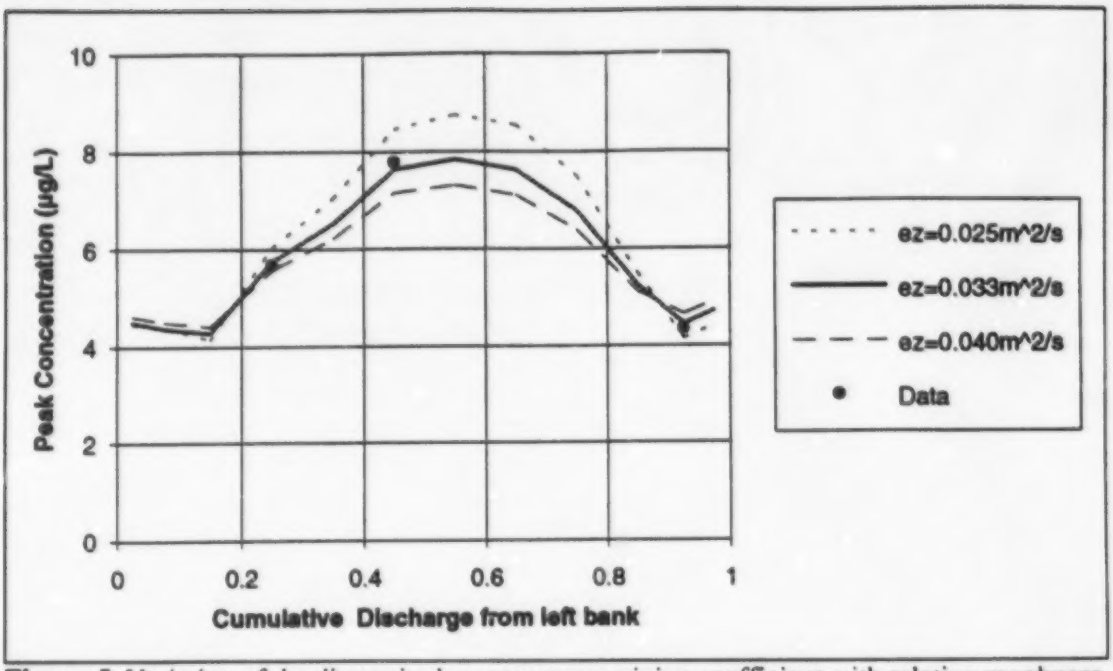

Figure 5 Variation of the dimensionless transverse mixing coefficient with relative roughness.

Dykema, 1991):

[17] $B = 115\,Q^{0.09}$

[18] $H = 0.13\,Q^{0.48}$

and

[19] $U = 0.07\,Q^{0.43}$

are used to rearrange equation [16] in terms of k_t, the result is

[20] $k_t = 0.018\,Q^{0.52}$

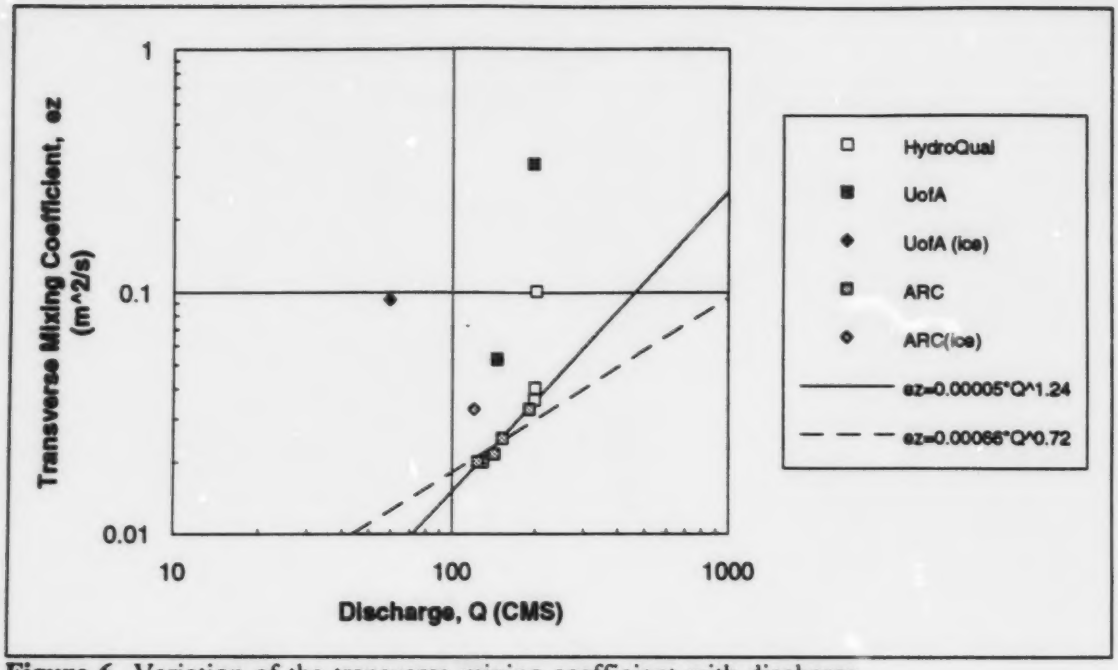

Figure 6 Variation of the transverse mixing coefficient with discharge.

which indicates that k_t is not independent of discharge. However, when this equation is compared to the data set in figure 7 it can be seen that a constant value of k_t would fit the data just as well. This lack of a distinct trend indicates that it may be just coincidence that equation [16] fit the data so well.

Assuming that the trend described by equation [16] is correct, the dimensionless diffusion factor, β can also be described as a function of discharge. Equations [16], [17], and [19] can be combined to produce the following equation

$$[21] \qquad \beta = 0.000014\, Q^{0.72}$$

The data shown in figure 8 does follow this trend somewhat, but the deviations of the data from this line are no less than they are from the mean value of the data subset. Additional mixing experiments are needed to establish whether or not the apparent variations described above are

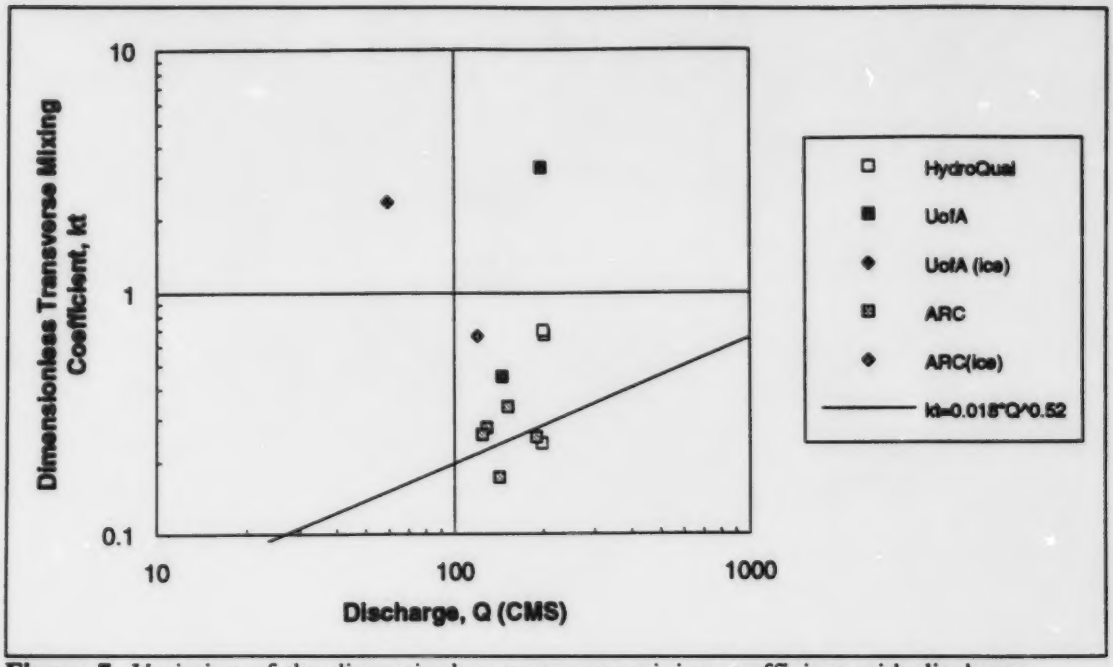

Figure 7 Variation of the dimensionless transverse mixing coefficient with discharge.

real.

The above analysis shows a possible trend in the data but it is just as accurate, in most cases, to use a mean value of k_t of 0.25 or a mean value of β of 0.0004 to describe the mixing rate. However, both these parameters cannot be constant at the same time. Equation [15] indicates that if one of the parameters is held constant, the other would be a function of aspect ratio and relative roughness.

It would be ideal if one of the dimensionless parameters could be shown to be a constant that is independent of the flow conditions. This would make it much easier to evaluate transverse mixing for alternate flow conditions. Lau and Krishnappan (1977) suggest that a variation of β was the better parameter to describe transverse mixing because they found it to vary with the river aspect ratio. However, this variation indicates that it is more likely that the parameter k_t is the constant.

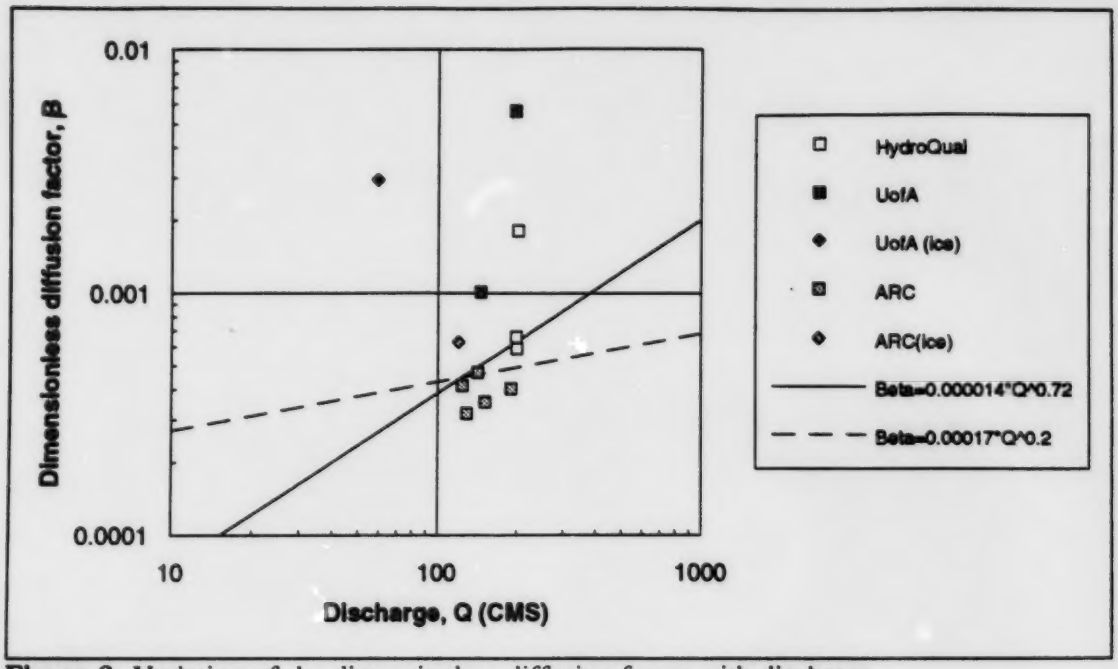

Figure 8 Variation of the dimensionless diffusion factor with discharge.

If the dimensionless transverse mixing coefficient, k_t is assumed to be independent of discharge, the mixing coefficient e_z can be shown to be a function of discharge by substituting Manning's Equation into equation [14]. This results in the expression

$$[22] \qquad e_z = \frac{k_t n \sqrt{g}}{H^{1/6}} \frac{Q}{B}$$

where g is the acceleration due to gravity, n is the Manning roughness and the hydraulic parameters H, B, and n are either constant or functions of discharge. When the typical hydraulic characteristics for the reach downstream of Edmonton described by equations [17],[18] and [19] are substituted in equation [22] and k_t is assumed to be a constant of 0.25, the result is

[23] $e_z = 0.00066\,Q^{0.72}$

This equation does not fit the data as well as the best fit function given by equation [16], but it still follows the trend exhibited by the data (figure 8)

The dimensionless diffusion factor, β can also be written as a function of the hydraulic conditions by substituting Manning's Equation into equation [15]. The result is

[24] $\beta = \psi m_x k_t n \sqrt{g} \frac{H^{5/6}}{B}$

When the Leopold-Maddock relationships are used to define the hydraulic conditions for the reach downstream of Edmonton, the following relationship for β is obtained

[25] $B = 0.00017\,Q^{0.2}$

This indicates that β is only a weak function of discharge. The variation of β predicted by equation [25] over the range of test discharges is less than the scatter in the actual data (figure 8). This scatter in the data set does makes it difficult to justify using either equation [21] or [25] to predict β for low flow conditions, even though the previous analysis indicates that β will increase with increasing discharge. Since no improvement in modelling accuracy can be gained by using a dimensionless diffusion factor which varies with discharge, it is recommended that $\beta = 0.0004$ be used modelling when discharges are less than 200 m^3/s and when sinuosity is not a factor.

No definitive statement can be made about the value of β at discharges higher than 200 m^3/s until additional information is obtained for high flows. The two relationships developed to predict β, equations [21] and [25], give very different values at higher flows. For example, at

a discharge of 500 m^3/s, equation [21] gives $\beta = 0.0012$ while equation [25] gives $\beta = 0.0006$. These values will produce different results when used in a model. Additional experiments performed at higher flows will provide the additional information required to determine the actual variation of β over a wide range of discharges.

4.2.2 Location

The location of a river reach, as well as the discharge, affects the rate of transverse mixing. As can be seen in equations [22] and [24], the width, depth and hydraulic roughness of a river reach affects the value of both e_z and β. The Leopold-Maddock equations, which describe the variations of width and depth with discharge, vary with both hydraulic roughness and slope, therefore the coefficients in both equation [23] and [25] should vary from reach to reach.

Location also determines the sinuosity of a river reach and there is some indication that sinuosity affects the mixing coefficient. The two highest values of mixing coefficient were both from tests reaches consisting of one bend in a river. The rest of the tests were performed in reaches with little or no sinuosity so no explicit relationship could be established between the two parameters (Sinuosity can be defined as the ratio between the thalweg and down valley distances). Bends may produce an apparent increase in e_z because, in addition to the turbulent mixing, some mixing is caused by the secondary currents. The streamtube approach eliminates the effects of transverse and vertical velocity components introduced by changes in the cross-section shape but cannot account for the transport of mass produced by the three-dimensional helical motion in a bend. Therefore, transverse mixing coefficients evaluated in sinuous reaches should not be used to model mixing in straighter reaches. Conversely, higher than average values of e_z should be used to model mixing in highly sinuous reaches.

4.1.3 Ice Cover

The effects of an ice cover on the mixing in natural channels has not been satisfactorily resolved at this time (Elhadi et al., 1984). In a straight laboratory channel, Engmann (1974)

found that the presence of a top cover reduced e_z to about one-half of its open water value. The dimensionless mixing coefficient would not change, however, if it were defined in terms of the hydraulic radius, R instead of the flow depth, H. Data from ice-covered natural channels obtained by Beltaos (1978) do not support this conclusion. Dimensionless mixing coefficients for the natural channels were much more consistent when the flow depth was used in the ratio.

The data shown in figure 6 indicate that the presence of an ice cover tends to increase the transverse mixing coefficient for the same discharge in the same reach. This increase in mixing rate appears to be highly variable. More mixing experiments under ice conditions are needed to determine an explicit change in e_z due to the presence of an ice cover, especially since this meagre data contradicts the results of previous studies.

5.0 SUMMARY AND RECOMMENDATIONS

More rigorous regulation and monitoring of effluent discharges in Alberta has created a demand for additional information on the variation of the transverse mixing coefficient with changes in the hydraulic characteristics in the North Saskatchewan River. A number of mixing studies were done in this river in the past; a search of existing data uncovered twelve tracer dye studies with sufficient information to evaluate the transverse mixing coefficient. Two analytical techniques and three numerical models were used to evaluate transverse mixing coefficients ranging between 0.02 m^2/s and 0.34 m^2/s from both steady state and time varying tracer dye experiments at river discharges ranging between 59 m^3/s and 202 m^3/s.

There are limitations to each of the test procedures and methods of analysis used evaluate the mixing coefficients given in this report. The results of the steady state tests are usually very site specific and usually only cover short reaches. Also, because of the injection equipment involved, the injection sites tend to be near the banks where there is greater variability in transverse mixing due to changes in channel shape. The difficulty with unsteady experiments is that it not only requires sufficient sampling across the channel but also requires sampling over time as the dye cloud passes the sample site. Analytical evaluations usually assume uniform hydraulic characteristics over the test reach so the error increases as the channel variability increases. Input data to numerical models must also satisfy certain numerical stability criteria to obtain valid solutions which means that either considerable effort is required to define the channel characteristics or much of the channel shape must be interpolated from the available data. Also, the transverse mixing coefficients used to calibrate numerical models may include effects of the numerical formulation or grid spacing. Therefore, no matter which method is used to evaluate the transverse mixing coefficient there will be some uncertainty in the result.

In addition to the limitations for numerical models in general, there are some limitations to each of the three numerical models described in this report. A limitation of the MULTI model is that it uses the mean velocity of the river in all the stream tubes so it does not predict the lag of the near bank streamtubes compared to the centerline streamtubes. This may produce a

significant source of error when concentration distributions from multiple effluent sources are superimposed. One of the limitations of the ARC model is that the amount of computing power required increases exponentially with river length. This means that there is a practical limit to the reach lengths that can be modelled using this approach. Another limitation of the existing model is that it has not been designed to accept time-varied concentration inputs. Also, the ARC model does not have an effluent decay term at this time. The major limitation of the TRSMIX model is that it can only be used for steady state injections.

Two dimensionless parameters were identified: the dimensionless transverse mixing coefficient, k_t and the dimensionless diffusion factor, β. Both these parameters showed no significant correlation with hydraulic characteristics for data reported in the literature. However, a trend was identified between the actual mixing coefficient and the discharge for the lower bound data from the North Saskatchewan River downstream of Edmonton. Relationships between the dimensionless parameters and discharge were derived from this trend using the Leopold-Maddock relationships which describe the variation of the hydraulic characteristics with discharge. However, it was found to be just as accurate, for the range of discharges tested, to use mean values of $k_t = 0.25$ or $\beta = 0.0004$ to describe the mixing rate. Both parameters can not be constant at the same time so, if k_t is assumed constant, a second relationship between β and discharge is obtained. Additional mixing experiments at higher discharges are needed to determine which of these three relationships between β and discharge best describes the variation in the data.

The data indicates that the river slope and hydraulic roughness of a river reach play a role in determining the value of the transverse mixing coefficient. As well, reaches with high sinuosity tend to have much higher mixing rates. The data also seemed to indicate that the presence of an ice cover increased the transverse mixing coefficient which is contrary to the results in the literature.

The analysis indicates that $\beta = 0.0004$ should provide reasonable estimates of the transverse mixing in the North Saskatchewan River downstream of Edmonton for discharges

between 100 m^3/s and 200 m^3/s. For higher discharges or other reaches, equation [24] can be used to estimate the dimensionless diffusion factor; however, less confidence should be placed in the values from this equation. These same values may be used for ice-covered conditions as well, however, the results may not be as accurate as for open water conditions. Also, the mixing coefficient should be adjusted upwards for the values given by equations [24] if the reach is highly sinuous. Finally, if a numerical model is being used to predict concentration distributions, the model should be compared to the some test data to ensure that the estimated coefficients are appropriate for the particular model.

Additional experiments should be performed to measure transverse mixing under an ice cover to clarify the effects that an ice cover has on the transverse mixing coefficient. As well, additional open water experiments at high discharges should be done if modelling of transverse mixing is required for discharges greater than about 400 m^3/s.

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